

## FROM QCD TO DUAL SUPERCONDUCTIVITY TO EFFECTIVE STRING THEORY

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We show how an effective field theory of long distance QCD, describing a dual superconductor, can be expressed as an effective string theory of superconducting vortices. We use the semiclassical expansion of this effective string theory about a classical rotating string solution in any spacetime dimension  $D$  to obtain the semiclassical meson energy spectrum. We argue that the experimental data on Regge trajectories along with numerical simulations of the heavy quark potentials provide good evidence for an effective string description of long distance QCD.

### 1. From QCD to Dual Superconductivity

In the dual superconductor mechanism of confinement<sup>1,2,3</sup> a dual Meissner effect confines color electric flux to narrow tubes<sup>4</sup> connecting a quark-antiquark pair. In the confined phase, monopole fields  $\phi$  condense to a value  $\phi_0$ , and dual potentials  $C_\mu$  acquire a mass  $M = g\phi_0$  via a dual Higgs mechanism. The dual coupling constant is  $g = 2\pi/e$ , where  $e$  is the Yang-Mills coupling constant. Quarks couple to dual potentials via a Dirac string connecting the quark-antiquark pair along a line  $L$ , the ends of which are sources and sinks of color electric flux. The color field of the pair destroys the dual Meissner effect near  $L$  so that  $\phi$  vanishes on  $L$ . At distances transverse to  $L$  greater than  $1/M$  the monopole field returns to its bulk value  $\phi_0$ , so that the color field is confined to a tube of radius  $a = 1/M$  surrounding the line  $L$ . As a result, for quark-antiquark separations  $R$  greater than  $a$ , a linear potential develops that confines the quarks in hadrons. The string tension  $\sigma \sim \phi_0^2$ , so that  $M \sim \sqrt{\sigma/\alpha_s}$ . (The running coupling  $\alpha_s$  is evaluated at a scale of order  $M$ .)

Recent lattice calculations<sup>5</sup> and general arguments based on the work of 't Hooft<sup>6</sup> show that the confined phase of non-Abelian gauge theory is characterized by a dual order parameter (monopole condensate), which vanishes in regions of space where electric color flux can penetrate and

"dual superconductivity" is destroyed. This provides a basis in QCD for a generic dual superconducting effective field theory of long distance Yang Mills theory in which the dual gluon mass  $M$  serves as the ultraviolet cutoff.

To obtain an effective string theory of long distance QCD we do not need a specific form for the action  $S[C_\mu, \phi]$  of the effective dual field theory. This theory must have classical  $Z_N$  electric vortex solutions, and it cannot have massless particles. For  $SU(N)$  Yang Mills theory this can be done by coupling dual non-Abelian potentials  $C_\mu$  to  $Z_N$  invariant dual Higgs fields  $\phi$ . (For example there could be Higgs fields in the adjoint representation of the gauge group.) The long distance effective dual theory will then have the same symmetries as  $SU(N)$  Yang Mills theory and the dual  $SU(N)$  gauge symmetry can be "spontaneously broken" so that the gauge bosons  $C_\mu$  all acquire a mass, and there are  $Z_N$  electric flux tube excitations.<sup>7</sup>

In the classical approximation to the dual theory the axis of the flux tube is a straight line between the quark and the antiquark. The contribution of flux tube fluctuations to the heavy quark potential is determined by the path integral over all field configurations  $C_\mu, \phi$ , for which the monopole fields  $\phi$  vanish on some line  $L$  connecting the quark-antiquark pair.<sup>8</sup> The fluctuating vortex line  $L$  sweeps out a fluctuating spacetime surface  $\tilde{x}^\mu$ , whose boundary is the loop  $\Gamma$  formed by the worldlines of the moving pair. These surfaces  $\tilde{x}^\mu$  determine the location of the vortices, where dual superconductivity is destroyed and electric color flux can penetrate.

The Wilson loop  $W[\Gamma]$  of Yang Mills theory determining the quark-antiquark interaction is the partition function of the dual theory in the vortex sector.<sup>8</sup>

$$W[\Gamma] = \int \mathcal{D}C_\mu \mathcal{D}\phi \mathcal{D}\phi^* e^{iS[C_\mu, \phi]}. \quad (1)$$

The path integral (1) goes over all field configurations for which the monopole field  $\phi(x)$  vanishes on some sheet  $\tilde{x}^\mu$  bounded by the loop  $\Gamma$ .

## 2. From Dual Superconductivity to Effective String Theory

We transform the field theory partition function (1) to a path integral over the vortex sheets  $\tilde{x}^\mu$ , so that  $W[\Gamma]$  takes the form of the partition function of an effective string theory of these vortices. We do this in two stages:

- (1) We integrate over all field configurations  $C_\mu, \phi$ , containing a vortex located on a particular surface  $\tilde{x}^\mu$ , where  $\phi(\tilde{x}^\mu) = 0$ . This integration determines the action  $S_{\text{eff}}[\tilde{x}^\mu]$  of the effective string theory.

- (2) We integrate over all vortex sheets  $\tilde{x}^\mu(\xi)$ ,  $\xi = \xi^a$ ,  $a = 1, 2$ . This integration goes over the amplitudes  $f^1(\xi)$  and  $f^2(\xi)$  of the two transverse vortex fluctuations in a particular parameterization of the world sheet,  $\tilde{x}^\mu(\xi) = x^\mu(f^1(\xi), f^2(\xi), \xi)$ , and gives <sup>9</sup>

$$W[\Gamma] = \int \mathcal{D}f^1 \mathcal{D}f^2 \Delta_{FP} e^{iS_{\text{eff}}[\tilde{x}^\mu]}, \quad (2)$$

where

$$\Delta_{FP} = \text{Det} \left[ \frac{\epsilon_{\mu\nu\alpha\beta}}{\sqrt{-g}} \frac{\partial x^\mu}{\partial f^1} \frac{\partial x^\nu}{\partial f^2} \frac{\partial \tilde{x}^\alpha}{\partial \xi^1} \frac{\partial \tilde{x}^\beta}{\partial \xi^2} \right] \quad (3)$$

and  $\sqrt{-g}$  is the square root of the determinant of the induced metric

$$g_{ab} = \frac{\partial \tilde{x}^\mu}{\partial \xi^a} \frac{\partial \tilde{x}_\mu}{\partial \xi^b}. \quad (4)$$

The partition function (1) of an effective field theory has been expressed as the partition function (2) of an effective string theory. The presence of the determinant  $\Delta_{FP}$  in (2) makes the path integral invariant under reparameterizations  $\tilde{x}^\mu(\xi) \rightarrow \tilde{x}^\mu(\xi(\xi'))$  of the vortex worldsheet.

The parameterization invariant measure in the path integral (2) is universal and is independent of the explicit form of the underlying field theory. On the other hand, the action  $S_{\text{eff}}[\tilde{x}^\mu]$  of the effective string theory is not universal, and depends upon parameters in the action  $S[C_\mu, \phi]$  of the effective field theory describing the dual superconductor. However, for wavelengths  $\lambda$  of the string fluctuations greater than the flux tube radius  $a = 1/M$ , which are those included in (2), the action  $S_{\text{eff}}[\tilde{x}^\mu]$  can be expanded in powers of the extrinsic curvature tensor  $\mathcal{K}_{ab}^A$  of the sheet  $\tilde{x}^\mu$ ,

$$S_{\text{eff}}[\tilde{x}^\mu] = - \int d^4x \sqrt{-g} \left[ \sigma + \beta (\mathcal{K}_{ab}^A)^2 + \dots \right]. \quad (5)$$

The extrinsic curvature tensor is

$$\mathcal{K}_{ab}^A = n_\mu^A(\xi) \frac{\partial^2 \tilde{x}^\mu}{\partial \xi^a \partial \xi^b}, \quad (6)$$

where  $n_\mu^A(\xi)$ ,  $A = 1, 2$  are vectors normal to the worldsheet at the point  $\tilde{x}^\mu(\xi)$ . The values of the coefficients in this expansion ( the string tension  $\sigma$ , the rigidity  $\beta$ , ...) are determined by the parameters of the underlying effective field theory, (For example in a dual superconductor on the border between type I and type II the rigidity vanishes.) <sup>10</sup> If these coefficients are taken as parameters, the specific form of the dual field theory does not enter explicitly in the effective string theory of dual superconductivity (2).

The expansion parameter in (5) is the ratio  $(a/\lambda)^2$  of the square of the flux tube radius to the square of the wave length of the string fluctuations, and the leading term in this expansion is the Nambu–Goto action.

### 3. The Heavy Quark Potentials

Expanding the action (5) in small fluctuations about a straight string connecting two static quarks puts the partition function (2) in the form used by Lüscher, Symanzik and Weisz<sup>11,12,13</sup> to calculate the contribution of string fluctuations to the static heavy quark potentials. The leading long distance expression for the energy levels  $E_n(R)$  of a fixed string of length  $R$ , vibrating in  $D$  dimensional spacetime is<sup>13</sup>

$$E_n(R) = \sigma R + \left(-\frac{D-2}{24} + n\right) \frac{\pi}{R}. \quad (7)$$

Corrections to (7) arise from the higher order terms in  $S_{\text{eff}}[\tilde{x}^\mu]$ , which give contributions proportional to powers of  $(a/R)^2$ . For  $n = 0$ , (7) reduces to the Lüscher ground state heavy quark potential.

Recent numerical simulations<sup>13,14</sup> provide striking confirmation of the Lüscher potential for quark-antiquark separations greater 0.5 fm. At this distance and (with  $D = 4$ ) the ratio  $\pi/(12\sigma R^2)$  of the leading semiclassical correction in eq. (7) (with  $n = 0$ ) to the classical term  $\sigma R$  is already small ( $\sim 0.2$ ). Thus the string behavior of the static potential sets in at a distance where the semiclassical expansion parameter is small.

The excited potentials  $E_n(R)$ ,  $n > 0$ , involve wave lengths of order  $R/n$ . When this wave length is of order of the flux tube radius, higher order terms in the effective action should become important and modify the string behavior of the excited potentials. Therefore as  $n$  increases, the distance  $R$  for which (7) should be applicable becomes larger. This expectation is in qualitative agreement with the recent lattice measurements of Juge, Kuti, and Morningstar<sup>15</sup> of the excited energy levels. They find that, although the excited potentials do not have string behavior at  $R = 0.5$  fm, there is, for  $R \approx 2$  fm, a rapid rearrangement of excited levels towards the string ordering (7). There are questions to be clarified, but these measurements of excited heavy quark potentials provide further evidence for an effective string theory description of long distance QCD.

### 4. Meson Regge Trajectories

We can also use the effective string theory to calculate the Regge trajectories of light mesons by attaching massless scalar quarks to the ends of a

rotating string.<sup>16</sup> (We treat neither chiral symmetry breaking nor string breaking.) Consider a quark-antiquark pair rotating with uniform angular velocity  $\omega$  and separated by a distance  $R$ . (See Fig. 1). Calculating the

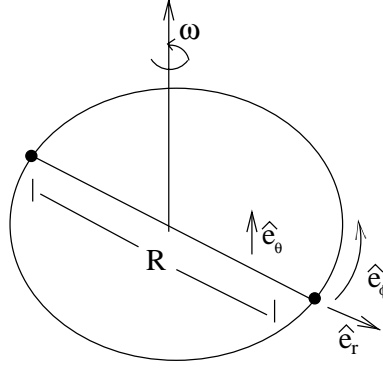


Figure 1. The string coordinate system

classical energy and angular momentum of a straight rotating string gives the usual linear Regge trajectories. To obtain the contribution of string fluctuations to these classical trajectories we expand the action  $S_{\text{eff}}[\tilde{x}^\mu]$  in small fluctuations  $f^i$  about a classical rotating straight string solution. In  $D$  spacetime dimensions there are  $D - 3$  fluctuations perpendicular to the plane of rotation and there is 1 fluctuation in the plane of rotation.

The semiclassical calculation of the path integral (2) around the classical rotating straight string solution with massless quarks contains an infrared divergence, so we introduce a quark mass as an infrared cutoff which can be set equal to zero only after renormalization. Furthermore, since the classical world sheet is no longer flat, in addition to the quadratic and linear ultraviolet divergences present in the calculation of the energy of the string fluctuations about a static string, the path integral (2) now contains a term which diverges logarithmically in the ultraviolet cutoff  $M$ . This divergence can be absorbed into a renormalization of a term in the boundary action called the geodesic curvature<sup>17</sup>.

The path integral (2) can then be evaluated for any values of the masses of the quarks on the end of the string and gives the energy levels of the interior of the rotating string for a prescribed motion of its ends. The frequency  $\omega$  determines the interquark distance  $R$ . These energy levels

reduce to the heavy quark potentials (7) if  $\omega$  is set equal to zero with  $R$  held fixed. For massless rotating quarks, these levels can be obtained from (7) by replacing the length  $R$  of the string by its proper length  $\pi/\omega$ , and by adding the term  $-\omega/2$ , which enters because the classical background is not flat. This gives<sup>16</sup>

$$E_n(\omega) = \frac{\pi\sigma}{\omega} + \left(-\frac{D-2}{24} + n\right)\omega - \frac{\omega}{2}. \quad (8)$$

To obtain the meson energy levels we must also quantize the motion of the quarks on the ends of the string.<sup>16</sup> For massless quarks, accounting for these boundary fluctuations effectively changes a Dirichlet boundary condition into a Neumann boundary condition. The meson energy levels are given by the same expression (8) and the meson angular momentum  $J$  is quantized,

$$J = l + \frac{1}{2}, \quad l = 0, 1, 2, \dots \quad (9)$$

The value of  $\omega$  is given as a function of  $J$  through the classical relation  $\omega = \sqrt{\pi\sigma/2J}$ . Squaring both sides of (8) and using the WKB quantization condition (9) yield the sequence of linear Regge trajectories,

$$E_n^2(l) = 2\pi\sigma \left( l - \frac{D-2}{24} + n \right), \quad n = 0, 1, 2, \dots \quad (10)$$

Corrections to (10) of order  $n^2/l$  come from the square of the term linear in  $\omega$  in (8) and from higher order terms in the semiclassical expansion.

Eq. (10), valid in any spacetime dimension, but applicable only for large angular momentum  $l$  gives, for  $n = 0$ , the contribution of string fluctuations to the leading Regge trajectory. The ratio  $(D-2)/24l$  of the leading semiclassical correction to the classical term is already small for  $l = 1$ . This could provide an explanation for the approximate experimental linearity of leading light meson Regge trajectories for  $l$  of the order 1, in analogy with the expectation from (7) that the string behavior of the static potential should set in at a distance  $R \sim 0.5$  fm.

The energies (10) (with  $n > 0$ ) of the excited states of the rotating string give rise to daughter Regge trajectories, and the calculation is applicable when  $l$  is much greater than  $n^2$ . According to this picture, linear daughter trajectories should be expected only for values of  $l$  much greater than 1. This corresponds to the expected linear behavior of the excited static potentials (7) only at values of  $R$  much greater than 0.5 fm.

## 5. Summary and Conclusions

- (1) We have found a path,  $\text{QCD} \rightarrow \text{Effective Field Theory of Dual Superconductivity} \rightarrow \text{Effective String Theory}$ , from QCD to effective string theory, which provides a concrete picture of the QCD string.
- (2) The derivation of the effective string theory made no use of the details of the effective dual field theory from which it was obtained, but the arguments<sup>5,6</sup> leading from QCD to an effective dual field theory description of long distance QCD need to be developed further.
- (3) The effective string theory provides an understanding of the values of the distances and of the angular momentum at which string behavior of physical quantities sets in.

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